

Bianchi I cosmology in the presence of a causally regularized viscous fluid

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We analyze the dynamics of a Bianchi I cosmology in the presence of a viscous fluid, causally regularized according to the Lichnerowicz approach. We show how the effect induced by shear viscosity is still able to produce a matter creation phenomenon, meaning that also in the regularized theory we observe the Universe emerges from a singularity with a vanishing energy density value. We discuss the structure of the singularity in the isotropic limit, when bulk viscosity is the only retained contribution. We see that, in the regularized theory we address, as far as viscosity is not a dominant effect, the dynamics of the isotropic Universe possesses the usual inviscid power-law behavior, but in correspondence of an effective equation of state, depending on the bulk viscosity coefficient. Finally, we show that in the limit of a strong non-thermodynamical equilibrium of the Universe, mimicked by a dominant contribution of the effective viscous pressure, a power-law inflation behavior of the Universe appears and the cosmological horizons are removed.

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Introduction

The initial cosmological singularity has been demonstrated to be a true, generic property of the Universe [1–3]. However, while the dynamics of the early Universe has been essentially understood, its physical and thermodynamical nature is far to be under control. On one hand, quantum gravity effects are able of altering the standard dynamical features proposed in [2, 4, 5], giving rise to fascinating alternatives (see for instance [6]) and particle creation effects can also be relevant [7, 8]. On the other hand, such an extreme region of evolution experiences a rapid expansion and non-trivial out-of-equilibrium phenomena become possibly important, including the emergence of viscous features in the cosmological fluid. More specifically, one usually distinguishes the bulk viscosity from the shear viscosity: while the former accounts for the non-equilibrium effects associated to volume changes, the latter is a result of the friction between adjacent layers of the fluid. As a matter of fact, shear viscosity does not contribute in isotropic cosmologies, whereas it may significantly modify the dynamics of an anisotropic Universe, as we shall see below. The simplest representation of a relativistic viscous fluid is provided by the so-called Eckart energy momentum tensor [9]. However, this formulation results to be affected by non-causal features and it actually predicts the propagation of superluminal signals [10]. In order to amend such a non-physical behavior, in [11] a revised approach has been proposed, solving the non-causality problem via the introduction of phenomenological relaxation times. Having in mind the basic role the Bianchi I cosmology has in understanding the generic behavior of an inhomogeneous Universe near the singularity, in [12] and [13] such a model is studied as sourced by a viscous fluid, in both the Eckart and in the revised approaches respectively. One of the most

intriguing issue coming out from such a study must be undoubtedly identified in the possibility of a singularity, from which the Universe emerges with negligible energy density and then a process of matter creation takes place. In the present analysis, we face the study of this afore-said peculiar solution in terms of an alternative causal regularization of the Eckart energy-momentum tensor, proposed by Lichnerowicz in [14]. Such a revised formulation is based on the introduction of the so-called *index of the fluid*, de facto a regulator scaling the four-velocity field, so defining a *dynamical velocity* of the fluid. This approach has been tested on some real systems, receiving interesting confirmation to its viability [15, 16]. However, being derived via a phenomenological approach, the Lichnerowicz energy momentum tensor must be completed by the specification of an ansatz linking the fluid index to the thermodynamical variables of the system, so closing the dynamical problem. In what follows, we implement the Lichnerowicz treatment to the viscous Bianchi I cosmology, by pursuing two different tasks. On one hand, we study the solution with matter creation, fixing the fluid index via the request of incompressibility. Results show that the Universe evolves through an intrinsic shear-driven anisotropic solution. This means that also in the Lichnerowicz scenario the solution with matter creation exists and, actually, such a phenomenon is enhanced, being therefore not related to non-physical effects of the Eckart formulation. On the other hand, we analyze the isotropic limit near the singularity, for which bulk viscosity is relevant. In this specific study we see that, as far as bulk viscosity is not dominant, the regularization provided by the index of the fluid preserves the same power-law behavior of the inviscid isotropic Universe. In other words, the bulk viscosity coefficient enters through an effective equation of state, ranging the same parameter domain of an ideal fluid (i.e. between dust and stiff matter).

Finally, we show how, if the bulk viscosity becomes sufficiently dominant, it is possible to get an equation of state having an effective polytropic index less than $2/3$ and then leading to a power-law inflation solution characterized by a massive entropy creation and no longer causal separation exists across the Universe regions.

Basic formalism

The Lichnerowicz original stress-energy tensor describing relativistic viscous fluids stands as follows [14]

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} - \left(\zeta - \frac{2}{3}\eta\right)\pi_{\mu\nu}\nabla_\alpha C^\alpha - \eta\pi_\mu^\alpha\pi_\nu^\beta(\nabla_\alpha C_\beta + \nabla_\beta C_\alpha), \quad (1)$$

where ρ is the energy density, p is the pressure, $g_{\mu\nu}$ denotes the metric tensor with the signature $(-+++)$ and u^μ is the four-velocity properly normalized as

$$u_\mu u^\mu = -1. \quad (2)$$

The bulk and shear viscous contributions are represented by the ζ and η coefficients, respectively. Here,

$$\pi_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu \quad (3)$$

is the projection tensor. Furthermore, C^μ represents the so-called dynamical velocity which is related to u^μ by

$$C^\mu = F u^\mu, \quad (4)$$

F being the index of the fluid.

A simple algebra shows that expression (1) can be rearranged as

$$T_{\mu\nu} = (\rho + p')u_\mu u_\nu + p'g_{\mu\nu} - \eta F [\nabla_\mu u_\nu + \nabla_\nu u_\mu + u_\mu u^\alpha \nabla_\alpha u_\nu + u_\nu u^\alpha \nabla_\alpha u_\mu], \quad (5)$$

where p' is the total pressure containing the standard thermodynamical contribution and the negative component due to viscosity, i.e.

$$p' \equiv p - \lambda \nabla_\alpha C^\alpha, \quad \lambda \equiv \zeta - \frac{2}{3}\eta. \quad (6)$$

The introduction of F was first due by Lichnerowicz in order to describe viscous processes in relativistic dynamics attempting to avoid superluminal signals. We can think of F as a contribution which eliminates the non-causality features of the Eckart's formulation by regularizing the velocity (F will be therefore referred below as the *regulator* of the theory). The cosmological consequences of the Lichnerowicz description in isotropic cosmologies have been examined in [16]. In [15]-[18], the index of the

fluid is parametrized as

$$F = \frac{p + \rho}{\mu}, \quad (7)$$

where μ is the rest mass-density which satisfies the conservation law

$$\nabla_\alpha(\mu u^\alpha) = 0. \quad (8)$$

The main advantages of the Lichnerowicz theory have been pointed out in [16]. A key-point consists in the fact that expression (5) reduces to the traditional description provided by Eckart upon setting $F = 1$. Furthermore, one of the main assumptions of the well-posedness theorems requires that [17, 18]

$$F > 1. \quad (9)$$

Lichnerowicz was lead to introduce a new formulation for viscosity first because of the study of incompressible perfect fluids for which the following relation holds

$$\nabla_\mu C^\mu = 0. \quad (10)$$

As we shall see below, this specific case is relevant near the singularity and actually it naturally fixes the behavior of the index of the fluid.

Field equations

The line-element of the Bianchi I model in the synchronous reference frame reads as

$$ds^2 = -dt^2 + R_1(t)^2 dx^2 + R_2(t)^2 dy^2 + R_3(t)^2 dz^2 \quad (11)$$

and hence the metric determinant is given by the following relation

$$\sqrt{-g} = R_1 R_2 R_3 \equiv R^3. \quad (12)$$

Above, x, y, z denote Euclidean coordinates and $R_1(t)$, $R_2(t)$ and $R_3(t)$ are dubbed cosmic scale factors. As well-known (see [3],[19] and [20]) such a model describes in vacuum an insitric anisotropic Universe, but in the presence of matter it can also admit the isotropic limit [21]. Let us now introduce the following quantity

$$H \equiv (\ln R) \cdot = \frac{1}{3} \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right). \quad (13)$$

where the dot denotes the time derivative. In order to have a compatible system, we set up the Einstein equations in a comoving frame with the matter source in which $u^0 = 1$ and $u^i = 0$, $i = 1, 2, 3$.

In this frame, $p' = p - \lambda\dot{F} - 3\lambda FH$ and the stress-energy tensor components are

$$\begin{aligned} T_0^0 &= -\rho, \\ T_i^i &= p' - 2\eta F \frac{\dot{R}_i}{R_i}. \end{aligned} \quad (14)$$

Thus, the Einstein equations for the Bianchi I spacetime, having the viscous Lichnerowicz tensor as source (here only the ii and 00-components are non-vanishing), can be written as

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} = -\chi \left(p' - 2\eta F \frac{\dot{R}_1}{R_1} \right), \quad (15)$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} = -\chi \left(p' - 2\eta F \frac{\dot{R}_2}{R_2} \right), \quad (16)$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} = -\chi \left(p' - 2\eta F \frac{\dot{R}_3}{R_3} \right), \quad (17)$$

$$\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} = \chi \rho, \quad (18)$$

χ being the Einstein constant.

From the spatial components (15,16,17), it is possible to show that the system admits the following integrals of motion

$$\frac{\dot{R}_i}{R_i} = H + s_i R^{-3} e^\varphi. \quad (19)$$

Here $\dot{\psi} \equiv -2\chi\eta F$ and the quantities s_i are such that $s_1 + s_2 + s_3 = 0$. The evolution of H is obtained using the trace of the ii -components combined with the 00-component (18) of the Einstein equation so getting

$$\dot{H} = \chi\rho - \frac{1}{2}\chi h - 3H^2 + \frac{3}{2}\chi\zeta FH + \chi\left(\frac{1}{2}\zeta - \frac{1}{3}\eta\right)\dot{F}, \quad (20)$$

where $h \equiv \rho + p$ represents the specific enthalpy. Moreover, the hydrodynamic equations $\nabla_\nu T_\mu^\nu = 0$ (only the 0-component is not vanishing here) provide the following evolution for ρ

$$\dot{\rho} = -4\chi\eta F\rho - 3Hh + 9H^2\zeta F + 12H^2\eta F + 3H\zeta\dot{F} - 2H\eta\dot{F}. \quad (21)$$

The first integrals (19) can be re-cast in a more compact form by the use of (20) and (18) as

$$\rho = \frac{1}{\chi} (3H^2 - q^2 R^{-6} e^{2\varphi}), \quad (22)$$

where $q^2 \equiv \frac{1}{2}(s_1^2 + s_2^2 + s_3^2)$. It worth noting that setting $q^2 = 0$ corresponds to the isotropic case. Equations (13),(20),(21) and (22) together with the ii -components represent the full set of the dynamical equations characterizing the present model. In order to close the system

we introduce a polytropic index γ and we consider an equation of state of the form

$$h = \gamma\rho, \quad 1 \leq \gamma \leq 2$$

where, e.g., $\gamma = 1$ corresponds to dust matter and $\gamma = \frac{4}{3}$ to the radiation cases, respectively.

Asymptotic solutions for an incompressible fluid

As well known [1], approaching the initial singularity, the Bianchi I solution in vacuum is Kasner-like and the presence of a perfect fluid is negligible. In [12], it has been shown instead that in the presence of a shear viscous contribution the situation significantly changes but a Kasner-like solution still exists and it is characterized by a vanishing behavior of the energy density. Here, we want to verify if such a peculiarity survives when the Eckart representation of the viscous fluid is upgraded in terms of a Lichnerowicz causal reformulation. It is rather easy to realize via a simple asymptotic analysis, that the ansatz in [15] fails in the region of small value of ρ . In fact, as predicted by such an ansatz, $F \sim \rho R^3$ and clearly vanishes near the singularity if $\rho \sim 0$ (as R^3 is going naturally to zero toward the singularity). In investigating the solutions in this region, we need therefore to search for a different representation of F and we infer its form by picking the case of an incompressible fluid. From equation (4) and by using the line-element (11), one immediately gets the following expression for F

$$F = \frac{F_0}{R^3}. \quad (23)$$

Clearly, F grows to infinity as approaching the singularity, without contradicting the Lichnerowicz request $F > 1$. Actually, Lichnerowicz points out that for a well-behaving solutions without superluminal signals, F is lower-bounded by unity and, in particular, we expect that the value of F should significantly grow in extremely relativistic regimes as near the cosmological singularities. As suggested in [12], we assume that in the region of low density the viscous coefficients can be expressed as power-laws of the energy density with exponents greater than unity, i.e.,

$$\eta = \eta_0 \rho^\alpha, \quad \zeta = \zeta_0 \rho^\beta \quad \alpha \geq 1, \beta \geq 1.$$

It is worth noting that for an incompressible fluid, defined by eq. (4), the ζ -terms automatically drop out from the equations and this is also the case treated in [12] because there the bulk viscosity contributions is asymptotically negligible, taking $\beta > \alpha$. In our analysis, in order to search for a consistent solution, we assume that the density vanishes faster than the volume R^3 as the system approaches the singular point $(H, \rho) = (+\infty, 0)$ in

the (H, ρ) plane¹. Moreover, in the right-handed side of equation (20) we can retain the quadratic term in H only. Then, the limiting form of the equations (20) and (21) are

$$\dot{H} = -3H^2, \quad (24)$$

$$\dot{\rho} = -3\gamma\rho H + 18\eta_0 F_0 \frac{\rho^\alpha}{R^3} H^2. \quad (25)$$

It is easy to see that $\dot{\varphi} \rightarrow 0$ as it contains a positive power-law of the energy density and hence we can fix, without loss of generality, $\varphi \equiv 0$. The constraint equation (22) reduces to the following relation

$$H = \sqrt{3}qR^{-3}. \quad (26)$$

Then, one can easily check that the leading order asymptotic solutions for $t \rightarrow 0$ of the simplified Einstein equations (24,25,28) read as

$$H = \frac{1}{3t}, \quad \rho = Kt^{\frac{2}{\alpha-1}}, \quad R^3 = \sqrt{3}qt, \\ R_\alpha = \left(\sqrt{3}qt\right)^{p_\alpha}, \quad p_\alpha = \frac{1}{3} + \frac{s_\alpha}{3q},$$

where K is a constant function of the model parameters. In order the solutions to be asymptotically self-consistent, the relations above are applicable only when $\alpha > 3$, slightly different with respect to the results given by the Eckart approach in [12] where $\alpha > 1$. It is worth noting that we found that the energy density in this case decays more rapidly to zero with respect to the non-regularized case studied in [12] by Belinskii and Khalatnikov where

$$\rho^{BK} \sim t^{\frac{1}{\alpha-1}}, \quad \alpha > 1. \quad (27)$$

In other words, we see how the Lichnerowicz approach leads to a cosmological model in which the universe emerges from the singularity with a greater matter-rate creation with respect to the Eckart case.

Asymptotic solutions for an isotropic universe

We now focus our investigation in the opposite regime where the energy density takes a diverging value near the singularity. Then, let us consider the isotropic limit of the solution by setting $q^2 = 0$ in the first integral (22) which reduces to

$$\rho = \frac{1}{\chi} 3H^2. \quad (28)$$

As a natural consequence of isotropy, shear viscosity is not permitted by the Einstein equations compatibility and bulk viscosity is the only retained contribution in the energy momentum expression (5). This set up has already been examined in [22] for an Eckart fluid via the dynamical system approach, and in [15] via the Lichnerowicz treatment in an attempt to explain dark-energy related current issues. Here we drew our attention to the behavior of the solutions in the limit of large density values. We address the problem by parameterizing F according to expression (7) or,

$$F = \frac{\gamma\rho R^3}{\mu_0}, \quad (29)$$

where we have used the fact that $\mu = \mu_0 R^{-3}$ because of the rest-mass conservation law (8). It is easy to check that the time derivative of F yields

$$\dot{F} = \left(3H + \frac{\dot{\rho}}{\rho}\right) F. \quad (30)$$

Then, in this limiting case, the Einstein equations take the form

$$\dot{H} = \chi \left(1 - \frac{1}{2}\gamma\right) \rho - 3H^2 + 3\chi\zeta FH + \frac{1}{2}\chi\zeta F \frac{\dot{\rho}}{\rho}, \quad (31)$$

$$\dot{\rho} = -3H\gamma\rho + 18H^2\zeta F + 3H\zeta F \frac{\dot{\rho}}{\rho}. \quad (32)$$

For the bulk viscosity coefficient in the limit of large density, we still have a power-law behavior of the type:

$$\zeta = \zeta_1 \rho^\beta, \quad 0 \leq \beta \leq \frac{1}{2}.$$

The solutions of the full-set of the field equations given by (13), (28), (31) and (32) are investigated assuming that the energy density and the volume are evolving like powers of time according to the relations

$$\rho = \rho_0 t^y, \quad R^3 = R_0^3 t^x, \quad y < 0, \quad x > 0, \quad (33)$$

as the time $t \rightarrow 0$. As a consequence, the viscosity evolves through the following expression

$$\zeta = \zeta_1 \rho_0^\beta t^{\beta y}. \quad (34)$$

From equation (33) with the use of (13) we immediately get

$$H = \frac{x}{3t}. \quad (35)$$

From equation (31) and being $\dot{H} \sim t^{-2}$, it is necessary that $y = -2$ in order the system to be self-consistent. Similar arguments lead us to conclude that

$$x = 2\beta + 1. \quad (36)$$

¹ For a detailed discussion of the qualitative dynamical behavior of the singular points in the Eckart representation see [12].

Then, one finds the following evolutions in terms of t for the energy density and the scale-factor

$$\rho = \frac{x^2}{3\chi} t^{-2}, \quad (37)$$

$$R = R_0 t^{\frac{2}{3\gamma_{eff}}}, \quad \gamma_{eff} \equiv \frac{2}{2\beta + 1}. \quad (38)$$

while γ is related to β via the following expression

$$\gamma = \frac{2}{2\beta + 1} \left[\frac{1}{1 - 4\beta(2\beta + 1)^\beta \frac{\zeta_1 R_0^3}{(3\chi)^\beta \mu_0}} \right]. \quad (39)$$

Furthermore, one can check that the regulator evolves with time as

$$F \sim t^{2\beta-1} \quad (40)$$

and it is asymptotically growing when $0 \leq \beta < 1/2$ and it is positive constant when $\beta = 1/2$, leading to a well-behaving regularized solutions. An interesting additional feature stands out from the behavior of the scale factor R . In the Friedmann universe in the presence of an ideal fluid the volume typically evolves as $R_{RW} \sim t^{\frac{2}{3\gamma}}$. Here we observe that we still have an isotropic limit, as it is found in [12], but instead of being negligible, the viscosity acquires a fundamental role in the dynamics of the Universe, driving the evolution through an effective equation of state. Indeed, the range of the possible values of R in (38) perfectly coincides with the standard non-viscous homogeneous and isotropic universe. In particular,

- for $\beta = 1/2$ we have $R \sim t^{2/3}$: the universe maps a dust-dominated Friedmann universe with an effective equation of state $\gamma_{eff} = 1$.
- For $\beta = 0$ we get $R \sim t^{1/3}$: the solution evolves toward a stiff-matter dominated Universe with an effective equation of state $\gamma_{eff} = 2$.

Thus, the introduction of F in the viscous Friedmann universe doesn't produce exotic effects, but instead it gives rise to an effective ideal fluid equation of state driven by the bulk viscosity.

Now we emphasize that by extrapolating our solutions to the regime $\beta > 1/2$ we obtain an intriguing dynamical property of the Universe. In fact, from equation (38) for $\beta = 1/2 + \varepsilon$ with $\varepsilon > 0$ we immediately get

$$\gamma_{eff} = \frac{1}{1 + \varepsilon} \rightarrow p = -\frac{\varepsilon}{(1 + \varepsilon)} \rho. \quad (41)$$

For $\varepsilon > 1/2$ (i.e. $\beta > 1$) this dynamical behavior corresponds to a powerlaw inflation solution, induced by a negative effective pressure $p < -1/3\rho$. Indeed, it is easy to realize that the cosmological horizon

$$d_h(t) \equiv R(t) \int_0^t \frac{dt'}{R(t')} \quad (42)$$

takes in this case a divergent value. In this scenario the Universe corresponds to a unique causal region and we can think of it as a viable solution to the horizon paradox. It is worth noting that, since we are dealing with an isotropic flat Universe, we get $H \sim \sqrt{\rho}$ (see equation (28)) and then the restriction above $\beta < 1/2$, derived for an Eckart representation of the fluid ($F \equiv 1$), acquires a clear physical meaning. In fact, as far as such a restriction holds, the negative effective pressure, due to bulk viscosity, behaves like $\rho^{\beta+1/2} < \rho$, for large ρ values. This means that the standard (positive) thermodynamical pressure $p = (\gamma - 1)\rho$ remains always the dominant contribution. This is coherent with the idea that the bulk viscosity representation of non-equilibrium effects is valid only on a perturbative level. Clearly, the presence of the regulator F slightly changes this situation since it enhances the weight of viscosity in the dynamics and this is at the ground of the present results. Nonetheless, the regime $\beta > 1$ can be qualitatively interpreted as a fluid-like representation for strong non-equilibrium effects, not surprising in the limit of the singularity, when the geometrical velocity of Universe collapse diverges. Thus, the power-law inflation solution we find in such an extreme regime can be thought as the qualitative feature induced by a cosmological continuous source, whose thermodynamical evolution can not be approximated via equilibrium stages. If we accept a fluid representation for such a limiting scenario, we can argue that the superluminal geometrical velocity of the early phases of the Universe expansion is able to open the horizon size, making the cosmological space causally connected as a whole.

We conclude this section by stressing a remarkable feature of the obtained solution which makes it essentially different from the same limit treated in [12]. Indeed, there, the isotropic solution was considered for $t \rightarrow 0_-$ simply because it corresponds to a singular point in the dynamical system approach of equations (13),(20),(21) and (22). Here we deal with increasing t values and our dynamics describes an expanding universe from the initial singularity. This choice characterizing the evolution regime is physically confirmed when one considers the behavior of the entropy per comoving volume. The latter has the form

$$\sigma \sim \rho^{\frac{1}{\gamma}} R^3 \sim t^{2\left(\frac{1}{\gamma_{eff}} - \frac{1}{\gamma}\right)}. \quad (43)$$

Since from equation (39) we see that

$$\gamma = \gamma_{eff} \left[\frac{1}{1 - 4\beta(2\beta + 1)^\beta \frac{\zeta_1 R_0^3}{(3\chi)^\beta \mu_0}} \right] > \gamma_{eff}, \quad (44)$$

it is straightforward to conclude that the entropy per comoving volume increases due to dissipation processes when the universe expands. To sum up, we are dealing with a cosmological paradigm in which the universe

emerges from the singularity causally connected as a whole and with a significant entropy creation (this effect is enhanced by increasing β values). This suggests how extreme non-equilibrium thermodynamics near the singularity could play a relevant role in solving some unpleasant paradoxes of the standard cosmological model, namely the horizon and entropy ones.

Conclusions

We have studied the influence of a viscous cosmological fluid on the Bianchi I Universe dynamics in the neighborhood of the initial singularity. The characterizing aspect of the presented analysis consists of describing the viscous fluid via the Lichnerowicz formulation, introducing a causal regulator (the index of the fluid) in order to ensure a causal dynamics. We have also pointed out how this additional new degree of freedom must be properly linked to the geometrical and thermodynamical variables. We have investigated the role of the two viscosity coefficients in two different limits, when only one of them is dynamically relevant. The shear viscosity has been taken into account in the case of an incompressible fluid, evaluating the modification that the regulator introduces in the well-known solution with matter creation derived in [12]. We have shown that such a peculiar phenomenon, characterized by an asymptotically vanishing energy density, not only survives in the Lichnerowicz formulation but it is actually enhanced. The presence of a bulk viscosity term has been analyzed in the isotropic limit of the Bianchi I cosmology and again asymptotically to the initial singularity. We derived a power-law solution, which outlines some interesting features: i) the standard non-viscous Friedmannian behavior is encountered when bulk viscosity is a small deviation from equilibrium ($\beta < 1/2$), but this time the fluid presents an equation of state with an effective dependence on viscosity; ii) when bulk viscosity fully dominates the dynamics with allowance made for strong non-equilibrium effects ($\beta > 1$), we see that the universe evolves through a power-law inflation solution to the initial singularity, implying the divergence of the cosmological horizon and the subsequent disappearance of the Universe light-cone. Entropy production has been addressed with particular reference to the isotropic limit and specifically for dominant viscosity, where entropy tremendously grows. Despite one may argue whether or not the above-mentioned fluid description is possible in this extreme regime of dominant bulk viscosity, the issues above strongly suggest that a comprehensive understanding of the Universe birth and of the so-called horizon and entropy paradox can not be achieved before a clear account of the non-equilibrium thermodynamical evolution near the singularity will be properly provided.

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